§4.1 Computing 4d central charges from 6d

definition (4d):

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{c}{16\pi^2}(\text{Weyl})^2 - \frac{a}{16\pi^2}(\text{Euler})$$

where

$$(\text{Weyl})^2 = R_{\mu\nu\rho\sigma}^2 - 2R_{\mu\nu}^2 + \frac{1}{3}R^3$$

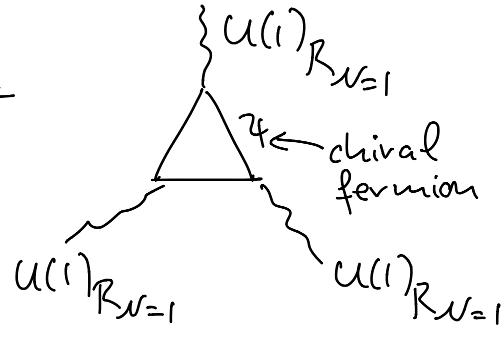
$$(\text{Euler}) = R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2$$

For $\mathcal{N}=1$ SCFT's they are related to $U(1)$ R-symmetry anomalies:

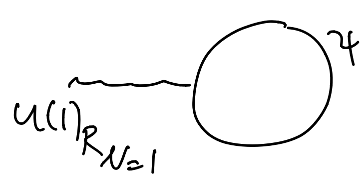
$$a = \frac{3}{32}\left[3\,\mathrm{tr}\,R_{\mathcal{N}=1}^3 - \mathrm{tr}\,R_{\mathcal{N}=1}\right],$$

$$c = \frac{1}{32}\left[9\,\mathrm{tr}\,R_{\mathcal{N}=1}^3 - 5\,\mathrm{tr}\,R_{\mathcal{N}=1}\right]$$

$\mathrm{tr}\,R_{\mathcal{N}=1}^3 =$



$\mathrm{tr}\,R_{\mathcal{N}=1} =$



't Hooft anomaly coefficients

Relation between anomaly coefficients and anomaly polynomial:

U(1) charge

$$I_6 = \frac{q^3}{6} c_1(F)^3 - \frac{q}{24} c_1(F) p_1(T_4)$$

U(1) bundle

anomaly polynomial of <u>one</u> Weyl fermion of charge q

$\longrightarrow$ summing over <u>all</u> Weyl fermions gives

$$I_6 = \frac{\operatorname{tr} R^3}{6} c_1(F)^3 - \frac{\operatorname{tr} R}{24} c_1(F) p_1(T_4)$$

In order to compute $\operatorname{tr} R^3$ and $\operatorname{tr} R$, we thus have to reduce I_8 of N M5-branes on Riemann surface Σ_g:

$$I_8(N) = N I_8(1) + (N^3 - N) \frac{p_2(N)}{24}$$

where

$$I_8(1) = \frac{1}{48}\left[p_2(N) - p_2(T) + \frac{1}{4}\left(p_1(T) - p_1(N)\right)^2\right]$$

Here, N and T stand for normal and tangent bundle of M5-branes on $\mathbb{R}^4 \times \Sigma_g$

$$p_1(B) = \sum_i b_i^2, \quad p_2(B) = \sum_{i<j} b_i^2 b_j^2$$

bundle   chern roots

twisting the (2,0) theory:

the 6d (2,0) theory has $Osp(6,2|4)$ superconformal symmetry

$\rightarrow$ bosonic subgroups:
- $USp(4) \simeq SO(5)$
  $\uparrow$
  R-symmetry
- 6d conformal group $SO(6,2)$

supercharges Q and scalar fields Δ transform under $SO(5,1) \times SO(5)_R$ as:

Q: $4 \otimes 4$ (with symplectic Majorana condition)

Δ: $1 \otimes 5$

two-form and spinors:

B_{MN}: singlet of $SO(5)_R$

ψ : $4' \otimes 4$

Put the theory on $\mathbb{R}^4 \times \Sigma_g$ and twist

$SO(2)_s$ of $\Sigma_g \hookrightarrow SO(5)_R$

$\uparrow$

spin connection

$\mathcal{N}=2$ twist:

$$SO(2)_R \times SO(3)_R \subset SO(5)_R$$
$$\updownarrow$$
$$SO(2)_S$$

Q transforms under $\underbrace{SO(3,1)}_{\mathbb{R}^4} \times \underbrace{SO(2)_S}_{\Sigma_g} \times SO(3)_R \times SO(2)_R$

as : $4 \otimes 4 \longrightarrow (2_{\frac{1}{2}} \oplus 2'_{-\frac{1}{2}}) \otimes (2_{\frac{1}{2}} \oplus 2_{-\frac{1}{2}})$

we twist as follows: $SO(2)_S \longrightarrow SO(2)_S - SO(2)_R$

$$Q \longrightarrow 2_0 \otimes 2_{\frac{1}{2}} \oplus 2_1 \otimes 2_{-\frac{1}{2}} \oplus 2'_{-1} \otimes 2_{\frac{1}{2}} \oplus 2'_0 \otimes 2_{-\frac{1}{2}}$$

preserved supercharges are: $2_0 \otimes 2_{\frac{1}{2}}$

$SO(2)_S$ singlet

$\longrightarrow \mathcal{N}=2$ superalgebra with $SU(2) \times U(1)$ R-sym

$U(1)_{R_{\mathcal{N}=2}} \simeq 2\, SO(2)_R$ due to $R[Q]=1$

$2'_0 \otimes 2_{-\frac{1}{2}}$ are conjugate supercharges Q^+

scalars Δ decompose as

$$1 \otimes 5 \longrightarrow 1_0 \otimes (3_0 \oplus 1_1 \oplus 1_{-1})$$

$$\xrightarrow{\text{twisting}} 1_0 \otimes 3_0 \oplus \underbrace{(1_{-1} \otimes 1_1)_{\mathbb{C}}}_{\text{complex scalar}}$$

$\mathcal{N}=1$ twist:

$$U(1)_R \times SU(2)_F \subset SU(2)\times SU(2)_F \simeq SO(4)\subset SO(5)_R$$

$\updownarrow$

$SO(2)_S$

Q transforms under $SO(3,1)\times SO(2)_S \times SU(2)_F \times U(1)$:

as : $$4\otimes 4 \longrightarrow \left(2_{\frac{1}{2}} \oplus 2'_{-\frac{1}{2}}\right)\otimes\left(2_0 \oplus 1_{\frac{1}{2}} \oplus 1_{-\frac{1}{2}}\right)$$

we twist $SO(2)_S \longrightarrow SO(2)_S - U(1)_R$

then

$$Q \to \left(2_{\frac{1}{2}}\otimes 2_0\right) + \left(2_0 \otimes 1_{\frac{1}{2}}\right) + \left(2_1 \otimes 1_{-\frac{1}{2}}\right) + \left(2'_{-\frac{1}{2}}\otimes 2_0\right) + \left(2'_{-1}\otimes 1_{\frac{1}{2}}\right) + \left(2'_0 \otimes 1_{-\frac{1}{2}}\right)$$

→ preserved supercharges: $2_0 \otimes 1_{\frac{1}{2}}$

(with conjugate $Q^{\dagger} = 2'_0 \otimes 1_{-\frac{1}{2}}$)

→ $\mathcal{N}=1$ SUSY

$R_{\mathcal{N}=1}$ is identified with $2q_{U(1)_R}$

scalars decompose as:

$$1\otimes 5 \to 1_0 \otimes \left(2_{\frac{1}{2}} + 2_{-\frac{1}{2}} + 1_0\right)$$

$$\xrightarrow{\text{twisting}} \underbrace{1_{-\frac{1}{2}}\otimes 2_{\frac{1}{2}}}_{\text{complex scalar}} + 1_{\frac{1}{2}}\otimes 2_{-\frac{1}{2}} + 1_0 \otimes 1_0$$

Now let us go back to the anomaly polynomial:

- Denote by $\pm\lambda_1, \pm\lambda_2, \pm t$ the Chern roots of the tangent bundle on $\underset{\lambda_1,\lambda_2}{\mathbb{R}^4} \times \underset{t}{\Sigma_g}$
- and by $\pm n_1, \pm n_2$ the Chern roots of the normal bundle
- Denote the $U(1)_R$ bundle by F

$\rightarrow$ $n_1 \rightarrow n_1 + c_1(F)$, $\quad n_2 \rightarrow n_2 + c_1(F)$

$\mathcal{N}=1$ supersymmetry requires

$$n_1 + n_2 + t = 0$$

Using $\int_{\Sigma_g} t = 2-2g$ and integrating over Σ_g, we get

$$\int_{\Sigma_g} I_8 = \frac{1}{6}(g-1)N^3 c_1(F)^3 - \frac{1}{24}(g-1)N c_1(F) p_1(T_4)$$

$\rightarrow$ $\mathrm{tr}\, R^3_{\mathcal{N}=1} = (g-1)N^3$, $\quad \mathrm{tr}\, R_{\mathcal{N}=1} = (g-1)N$